OP' IMALESTIMAT ION OF RAIN RAI EPROFILES F ROM RADAR RE T URNS AT AII ENUATING WAVE LENGTHS

Ziad S. tladdad and [astwood Im

Jet Propulsion l aboratory, California Institute of 'Technology
Pasadena, California

1. IN7 RODUCT ION

1 here are significant inherent ambiguities when one tries to determine a particular vertical rain intensity profile from a given time profile of radar echo powers measured by a downward- looking (space-borne or airborne) radar at a single attenuating frequency. In this paper, we quantify these ambiguities mathematically, and examine their effects on the performance of rain–rate retrieval algorithms initially proposed for use by the Precipitation Radar of the 'I ropical Rainfall Measuring Mission (TRMM). It turns out that by appropriately varying the parameters of the reflectivity–rain-rate (Z–R) and/or attenuation-rain-rate (K-R) power-law relationships, several substantially different hypothetical rain rate profiles can produce the same radar power profile. imposing the additional constraint that the path–averaged rain-rate be given reduces the ambiguities but still limits the utility of a combined single–frequency radar / single-channel radiometer system for vertical rain–rate profile retrieval. As was suggested by the work of others (e.g. Weinman et al. (1990), Marzoug and Amayenc (1991)), the problem of determining the rain attenuation coefficient profile (rather than the rain rate profile directly) from the single–frequency radar measurements turns out to be substantially less ambiguous. T hese results are more precisely quantified in section 2.

In sections 3 and 4, we suggest three potentail solutions to the ambiguity problem. '1 he first solution is to use dual-frequency radar measurements, Interestingly, the use of two frequencies does not remove the ambiguities in all theoretically possible situations, but, in practice, it gives a unique solution in almost all cases. The second solution uses additional measurements from a multi-frequency radiometer, which mimic the integrated rain-rate at several path lengths, to provide "boundary" conditions to further constain the estimates based on the radar measurements, '1 his multi--frequency --radiometer / single–frequency-radar data collection strategy will indeed be employed by the '1 RMM. Finally, in the case a single radar frequency is all that is available, the use of measurements collected at two distinct look angles can partially correct the ambiguity problem, provided that the spatial homogeneity of the two measurement sets is indeed preserved.

2. DETERMINISTIC AMBIGUITIES

let us begin by reviewing the classical result of Hitschfeld and Bordan (1954). We use the simple model that the power $p(r)$ received by a nadir–looking monostatic narrow-band radar, from range r, is proportional to the reflectivity coefficient η of the rain at range r, and to the inverse of the accumulated attenuation from range 0 to r. Calling $k(r)$ the attenuation coefficient at range r, and in the absence of noise, this is equivalent to assuming that the calibrated power is exactly given by

$$p(r) = \eta(r)\, e^{-\lambda \int_0^r k(t)dt} \qquad (1)$$

where $\lambda =$ 0.210 g(10). Suppose that we are given p and need to determine η and k. Following Hitschfeld and Bordan, it is empirically reasonable to assume that η and k are related by an equation of the form $\eta = \delta k^{\gamma}$, where the parameters δ and γ have to be determined within a certain range of possible values. We substitute this power-law in equation (1), and proceed to solve for k. As in Hitschfeld and Bordan (1954), the solution is

$$k(r) = \frac{p(r)^{1/\gamma}}{\delta^{1/\gamma} - \frac{\lambda}{\gamma}\int_0^r p(t)^{1/\gamma} dt} \qquad (2)$$

Since it is highly unlikely that 7 and δ can be known exactly, there remains to quantify the effect on k of an error in 7 or δ. Indeed, *any* value of (γ,δ) will give an attenuation profile k whose resulting radar returns would amount to the same given received power profile p. Specifically, for $k_1(r), \gamma_1$,61 to be produce the same power profile p as $k(r), \gamma, \delta$, it is necessary and sufficient that

$$k_1(r) = \frac{k(r)^{\gamma/\gamma_1} e^{-\frac{\lambda}{\gamma_1}\int_0^r k(t)dt}}{\left(\frac{\delta_1}{\delta}\right)^{1/\gamma_1} - \frac{\lambda}{\gamma_1}\int_0^r k(t)^{\gamma/\gamma_1} e^{-\frac{\lambda}{\gamma_1}\int_0^t k(\tau)d\tau} dt} \qquad (3)$$

An easy way to see this is to write down equation (2) for k_1 ,7$_1$, δ_1 , then substitute in it the expression for p

given by equation (l), using k, γ and δ. '1 bus, starting with a fixed rain scenario described by $k(r), \gamma, \delta$, and given any new values (γ_1, δ_1), we can concoct a new attenuation profile, using equation (3), that would have produced the same echo powers,

'1 he ambiguity is exacerbated if we try to determine the rain-rate profile itself directly from the data. Using power laws $\eta = aR^b$ and $k = \alpha R^\beta$, one finds that if we start with a fixed rain scenario described by $R(r), a, b, \alpha, \beta$, and given any new values $(a_1, b_1, \alpha_1, \beta_1)$, we can concoct a new rain-rate profile $R_1(r)$ that would have produced the same echo powers as R, namely that R_1 which satisfies

$$R_1(r) = \left(\frac{1}{\alpha_1} \frac{k(r)^\mu e^{-\nu \int_0^r k(t)dt}}{B - \nu \int_0^r k(t)^\mu e^{-\nu \int_0^t k(\tau)d\tau} dt} \right)^{1/\beta_1} \tag{4}$$

in which $B = (\alpha^{b/\beta} a_1/a)^{\beta_1/b_1}/\alpha_1$, $\mu = b\beta_1/(b_1\beta)$, $\nu = \lambda\beta_1/b_1$, and $k(r) = \alpha R(r)^\beta$.

If we use the surface-reference approach of Marzoug and Amayenc (1991), and if we start with a rain scenario described by $R(r), a, b, \alpha, \beta$, over a surface with backscattering coefficient σ_0, then given any new values $(a_1, b_1, \alpha_1, \beta_1)$ and σ_1, we can con-

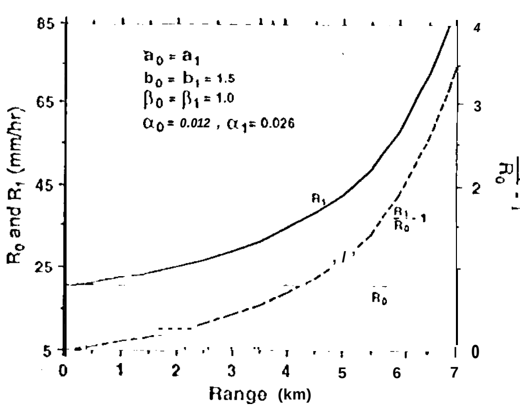

Figure 1: ambiguous profiles

coct a new rain–rate profile $R_1(r)$ that would have produced the same surface-referenced echo powers as R, namely that R_1 which satisfies

$$R_1(r) = \left(\frac{1}{\alpha_1} \frac{k(r)^\mu e^{\nu \int_r^{r_0} k(t)dt}}{B' + \nu \int_r^{r_0} k(t)^\mu e^{\nu \int_t^{r_0} k(\tau)d\tau} dt} \right)^{1/\beta_1} \tag{5}$$

where r_0 is the maximal range and B' is given by $B' = ((\sigma_0/\sigma_1)\alpha^{b/\beta} a_1/a)^{\beta_1/b_1}/\alpha_1$.

By varying the (many) parameters in equations (3), (4) or (5), one can easily produce different rain profiles that give the same echo power profile. Figure 1 shows the example of a constant rain rate profile R_0, and the ambiguous profile R_1 obtained by changing the value of α only. In this case, the relative error increases exponentially. One can similarly produce profiles that not only give the same echo power profile, but that have the same path--averaged rain-rate as well.

Before considering solutions to this problem, we note one additional important implication of the ambiguities problem, namely that one should be very careful in evaluating the performance of any particular rain-retrieval algorithm. Indeed, the conventional method of postulating a profile, simulating the resulting echo power data, running them through the algorithm of interest, then comparing its estimate with the original profile is very difficult to justify once one realizes that the particular data used could have been produced by a continuum of rain profiles, and that, therefore, the fact that one's algorithm can select one of these profiles (rather than any of the other, a priori equally possible, ones) is not in itself a measure of good performance, In fact, now that we can write down all the deterministic ambiguous profiles giving rise to the same data, it would be interesting to describe the likelihood of occurence of each one, given some reasonable assumptions about the physics governing the problem. With these concerns in mind, let us consider modifications to our problem that can make it less ambiguous.

3. RADAR-ONLY SCHEMES

It is natural to expect that if we could analyze the backscattered power at two distinct frequencies, the problem of estimating the rain profile would be significantly less ambiguous, It turns out that, while there exist families of two-frequency echo power profiles for which there still is a continuum of rain-rate solutions, the two-frequency problem is indeed ***generically*** uniquely invertible.

If one frequency is all that is available, and if one can scan the same volume of rain at two distinct angles, then the ambiguities can be reduced. We have derived the equations for the extent of the ambiguities inherent in such a dual look-angle scheme. In this case, if one also assumes that the path-averaged rain-rate is given, the upper bound for the relative error Q between any two profiles giving the same echo power along the two look angles turns out to be small. Figure 2 shows a plot of Q

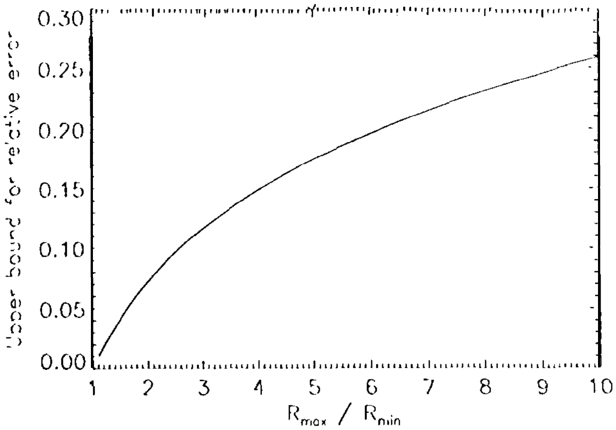

Figure 2: dual- look- angle relative error

as a function of the ratio R_{max}/R_{min} of the largest rain rate to the smallest one in the profile considered. As one can see from the graph, as long as $R_{max}/R_{min} \leq 5$ (a very realistic bound on the variation of the rain rate within a single "event"), the relative error cannot exceed 17% at any range.

4. RADAR + RADIOMETER

If radiometer measurements are available, one can try to fuse this additional data with the radar data. One way to do so is to calculate the joint probability density function $\mathcal{P}_r(R, c)$ for the rain rate R and the accumulated attenuation $c = \int_0^r R$, at all ranges r, conditioned on the radar data **only**, then,

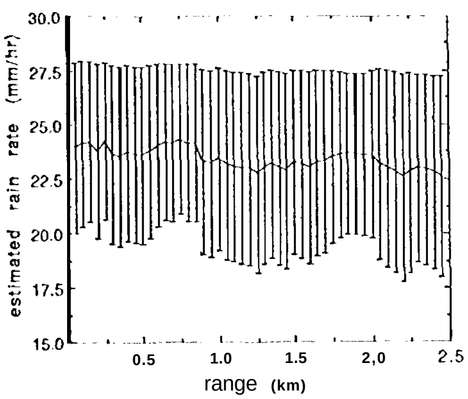

Figure 3: radar-only estimate (using $\mathcal{P}$)

using $\mathcal{P}$, calculate the conditional density $\mathcal{P}'$ for R given the additional radiometer measurements of c at various intermediate ranges. We have indeed carried out this approach, and an example of its results is shown in figures 3 and 4. For the simulated power profile from a constant rain rate of 20 mm/hr over 2.5 km, figure 3 shows a plot of the mean of $\mathcal{P}$ as a function of range, with error bars corresponding to $\pm$ one standard deviation of $\mathcal{P}$. Figure 4 shows the

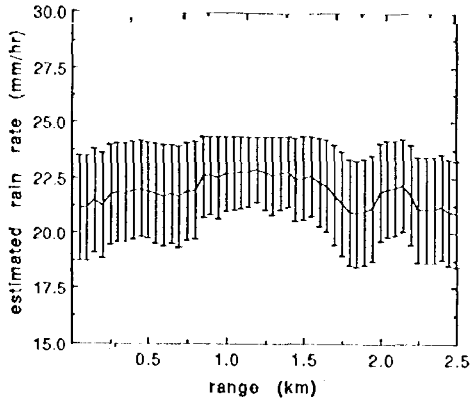

Figure 4: radar+radiometer estimate (using $\mathcal{P}'$)

corresponding plots for $\mathcal{P}'$ assuming a single 10 GHz radiometer channel. Even in this simple case, the approach quantifies the ambiguity in the radar-only estimate, and quantifies the improvement obtained using the radiometer.

5. ACKNOWLEDGMENTS

1 his research was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

6. REFERENCES

Brockett, R.W. 1981: Nonlinear Systems and Nonlinear Estimation Theory, *Stochastic Systems, M. Hazewinkel et. al., eds.*, 441-477.

Hitschfeld, W. and Bordan, J. 1954: Errors Inherent in the Radar Measurement of Rainfall at Attenuating Wavelengths, *J. Metero.*, 11, 58-67.

Marzoug, M. and Amayenc, P. 1991: Improved Range-Profiling Algorithm of Rainfall Rate from Spaceborne Radar with Path-integrated Attenuation Constraint, *I.E.E.E. Trans. Geo. Rem. Sen.*, 29-4, 584-592.

Weinman, J., Meneghini, R. and Nakamura, K. 1990: Retrieval of Precipitation F'refiles from Airborne Radar and Passive Radiometer Measurements, *J. Appl. Meteo.*, 29, 981-993.